

Foundations of Calculus Summer Assignment

The questions in this problem set are considered pre-requisite material for a student entering Calculus. All of the topics represented here were covered in prior classes and represent some of the techniques essential for success in this course. They are selected to reinforce important basic Pre-Calculus principles – mostly algebra, but some knowledge of functions in general.

Please show as much work as possible and be prepared to submit this assignment on the second day of class when we meet in September. *Your calculator is NOT allowed on these problems unless indicated.* When necessary, please round all answers to three decimal place accuracy.

Mr. Matte

1.) If $f(x) = x^3 - 2x - 1$ and $g(x) = 2x + 5$, please find the following:

a.) $f(g(-4))$

b.) $g(f(-4))$

2.) Consider the function $f(x) = \frac{1}{1-x^2}$. Please find:

a. $f(-2)$

b. $f\left(\frac{1}{2}\right)$

c. $f(a+h)$

3.) Please simplify the following expressions completely:

a.) $(5x^2 \cdot x^{-2})^2$

b.) $\frac{(-3x^{-3})^2}{-9x^{-4}}$

c.) $\frac{6x^{-2} + 9x^2}{3x^{-2}}$

d.) $\frac{x^{-1/2}(2x^{1/2} - x^{-1/2})}{x^{-1}}$

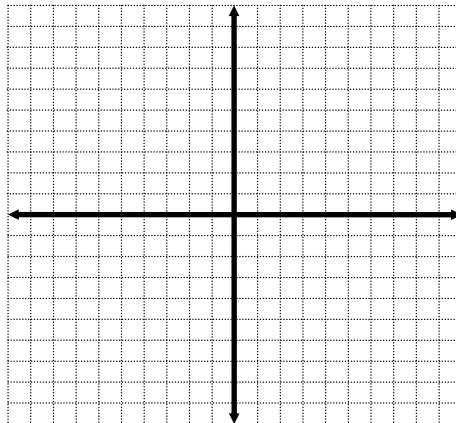
e.) $(5x^2 + x^{-2})^2$

f.) $\frac{(\sqrt{2x})^5}{(\sqrt{2x})^9}$

4.) Please use the *point-slope form of a line* to determine the equation of the line through the point $(7, -1)$ that is perpendicular to the line containing the points $(1, -3)$ and $(-2, 5)$.

5.) Please find the remaining roots of $f(x) = x^4 - 3x^3 - 14x^2 + 12x + 40$, given that $x = -2$ is double root.

6.) Please sketch the graph of $y = x(x+1)(x-2)^2$ on the axes below. Indicate the intercepts on the graph.



7.) Please solve the equation $7x^4 - 42x^2 = 35x$ for all values of x .

8.) Please determine the DOMAIN and the ROOTS of the following functions:

a.) $f(x) = \frac{x-3}{x^2-4}$

b.) $g(x) = \frac{\sqrt{x-2}}{x^2-x}$

c.) $h(x) = \frac{1}{\sqrt{x^2-4x}}$

9.) If $f(x) = x^3 + Ax^2 + Bx - 3$ and if $f(-1) = -6$ and $f(1) = 4$, what is the value of $2A + B$?

10.) Please find the inverse of the function $f(x) = 4\sqrt[3]{x-1}$

11.) Please solve the following equations using factoring methods:

a.) $x^2 - 64 = 0$

b.) $3x^2 + 2x - 1 = 0$

c.) $4x^2 - 21x - 18 = 0$

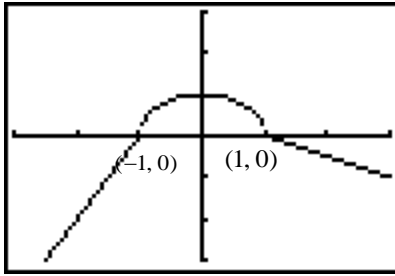
d.) $6x^3 + 3x^2 - 9x = 0$

12.) Find the intersection between $y = x^2 - 4x + 4$ and $y = 12 + 2x - x^2$ algebraically.

13.) Please use a graphing calculator to determine all solutions of the equation $x^3 - 4x^2 = 2x - 3$. (*No work needs to be shown, here. Just give the answers that you found on your calculator.*)

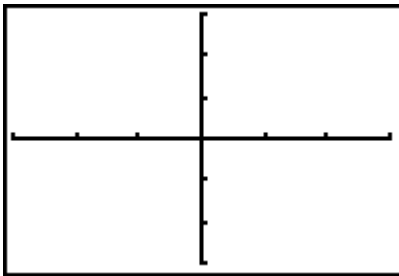
14.) Please solve the inequality $4x^3 - 21x^2 - 18x \leq 0$

15.) This is the graph of a function $y = f(x)$.

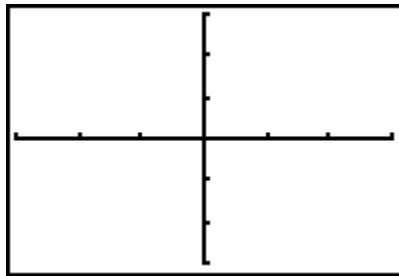


On the axes provided, please sketch the following:

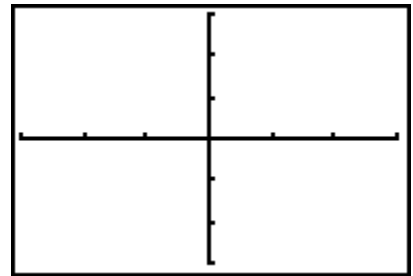
a.) $y = |f(x)|$



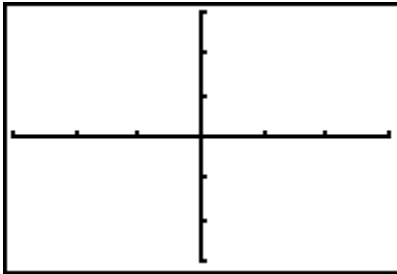
b.) $y = f(x+1)$



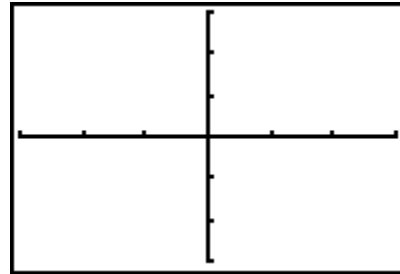
c.) $y = -2f(x)$



d.) $y = f(x) + 1$



e.) $y = -f(x-1) + 2$

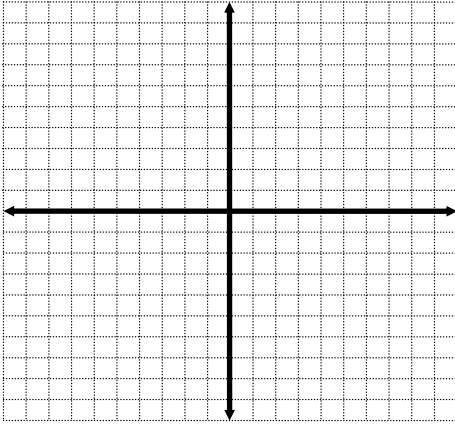


16.) a.) What are the domain and range of the function $f(x) = 4 - x^2$?

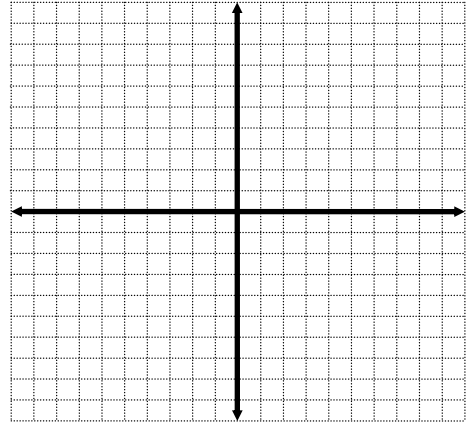
b.) What are the domain and range of the function $f(x) = 4 - x^2$, where $1 \leq x < 3$?

17.) On the axes provided, please sketch the graph of the following:

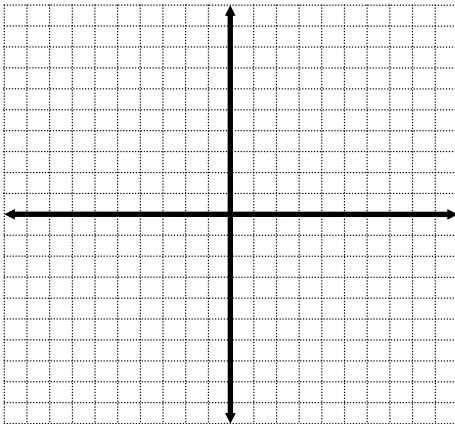
a.) $y = -x^2 + 4x - 4$



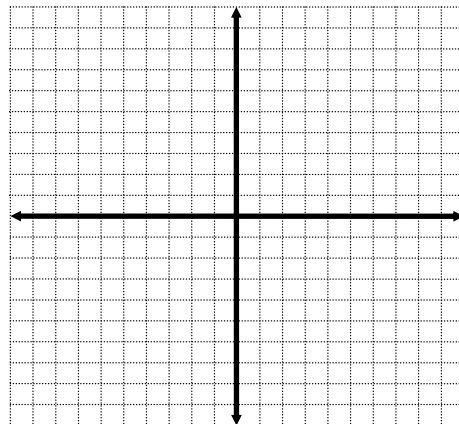
b.) $y = \begin{cases} 2x, & \text{if } x < -1 \\ 2x^2 + x - 3, & \text{if } -1 \leq x < 2 \\ -x + 3, & \text{if } x \geq 2 \end{cases}$



c.) $y = \sqrt{36 - x^2}$



d.) $y = a^x$, where $a > 1$



18.) Please solve each of the following for x : (Note: You may need to leave your answer in logarithmic or exponential form)

a.) $\log(x-2)=3$

b.) $\ln(x-2)=1$

c.) $2e^{2-x}+5=12$

d.) $\log_6(\log_2 x)=1$

e.) $9^x=4$

f.) $\ln x + \ln(2x+5) = \ln 7$

19.) Given a box of height h and whose base dimensions are x and y , please express the *surface area AND the volume* of the box, given the following conditions:

a.) The box is closed on all sides

b.) The box has an open top

c.) The box has an open top and a square base.

20.) A 6-foot tall person is standing x feet away from a 10 foot lamppost. What is the distance d from the base of the lamppost to the end of the person's shadow, expressed as a function of x ?

21.) A piece of wire 5 meters long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Please find an expression for the total area made up by the square and the circle as a function of x .

22.) Please determine the following (Remember No calculator!)

a.) $\cos 0$

b.) $\sin 0$

c.) $\tan \frac{\pi}{2}$

d.) $\cos \frac{5\pi}{6}$

e.) $\sec \frac{7\pi}{4}$

f.) $\tan 660^\circ$

g.) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

h.) $\tan^{-1}(-1)$

23.) Given the function $f(x) = 2\sin\left(\frac{2}{3}x\right) + 1$, please determine the

a.) amplitude

b.) range

c.) period

24.) Please circle the expressions that are identical:

$$\cos^2 x$$

$$(\cos x)^2$$

$$\cos(x^2)$$

$$1 + \sin^2 x$$

25.) Please circle the expressions that are identical:

$$(\sin x)^{-1}$$

$$\sin^{-1} x$$

$$\sin(x^{-1})$$

$$\frac{1}{\sin x}$$