

## *AP Calculus Summer Assignment*

The questions in this problem set are considered pre-requisite material for a student entering AP Calculus. All of the topics represented here were covered in prior classes and represent some of the techniques essential for success in this course. They are selected to reinforce important basic Pre-Calculus principles – mostly algebra, but some knowledge of functions in general. It is possible that you will look at many of these problems and say, “Hey, this is pretty easy. It’s just algebra. Why should I bother with it?” Trust me, you will hear me say many times this year – *Calculus is your friend, Algebra is your enemy*. Fight the enemy.

Please show as much work as possible and be prepared to submit this assignment on the second day of class when we meet in September. *Your calculator is NOT allowed on these problems (unless indicated – #11 & 14 only)*. When necessary, round all answers to three decimal place accuracy. Please have this assignment completed and ready to be submitted by Wednesday, September 7<sup>th</sup>.

**Mr. Matte**

1.) If  $f(x) = x^3 - 2x - 1$  and  $g(x) = 2x + 5$ , please find the following:

a.)  $f(g(-4))$

b.)  $g(f(-4))$

2.) If  $f(x) = \frac{1}{x}$ , please express  $\frac{f(x+h) - f(x)}{h}$  as a simplified rational expression.

3.) Please rationalize the denominator of the expression  $\frac{1}{\sqrt{x+h} + \sqrt{x}}$ . Simplify as much as possible.

4.) Please use the *point-slope form of a line* to determine the equation of the line through the point  $(7, -1)$  that is perpendicular to the line containing the points  $(1, -3)$  and  $(-2, 5)$ .

5.) Please factor the following polynomials:

a.)  $4x^2 - 21x - 18$

b.)  $3x^2 + 6x^3 - 9x$

c.)  $2x^3 + 3x^2 - 11x - 6$

d.)  $(x-1)^3(2x-3) - 2(x+6)(x-1)^2$

6.) Please solve the equation  $7x^4 - 42x^2 = 35x$  for all value of  $x$  .

7.) Please determine the DOMAIN and the ROOTS of the following functions:

a.)  $f(x) = \frac{x-3}{x^2-4}$

b.)  $g(x) = \frac{\sqrt{x-2}}{x^2-x}$

c.)  $h(x) = \frac{1}{\sqrt{x^2-4x}}$

8.) Please find the inverse of the function  $f(x) = 4\sqrt[3]{x-1}$

9.) If  $f(x) = x^3 + Ax^2 + Bx - 3$  and if  $f(-1) = -6$  and  $f(1) = 4$ , what is the value of  $2A + B$ ?

10.) Please simplify the quotient  $\frac{(x-1)^3(2x-3) - (4x-1)(x-1)^2}{(x-1)^2(2x-1)}$ . (*Suggestion: Factor the numerator.*)

11.) Please **use a graphing calculator** to determine all solutions of the equation  $x^3 - 4x^2 = 2x - 3$ . (*No work need be shown.*)

12.) Please solve the following equation for  $z$  :

$$2xy^3 + 3x^2y^2z + 4 = 3x^2y + x^3yz + 5z$$

13.) Please solve the inequality  $4x^3 - 21x^2 - 18x \leq 0$  (*No calculator, here. Sorry.*)

14.) Please *use your graphing calculator* to determine the values for which  $3^x \geq x^4$

15.) Please find the  $x$ -coordinate of the points at which the curves  $y - x + 1 = 0$  and  $2y + 3x^2 = 3x + y^2$  intersect.

16.) Please rewrite  $\frac{x^2 - 3x + 2}{\sqrt{x}}$  as a sum of terms with rational exponents.

17.) Please simplify the following expressions completely:

a.)  $(5x^2 \cdot x^{-2})^2$

b.)  $\frac{(-3x^{-3})^2}{-9x^{-4}}$

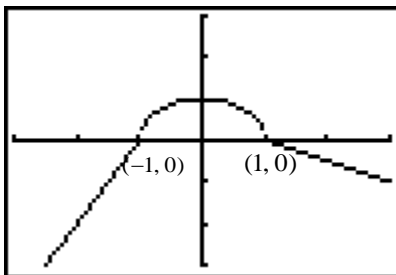
c.)  $\frac{6x^{-2} + 9x^2}{3x^{-2}}$

d.)  $\frac{x^{-1/2}(2x^{1/2} - x^{-1/2})}{x^{-1}}$

e.)  $(5x^2 + x^{-2})^2$

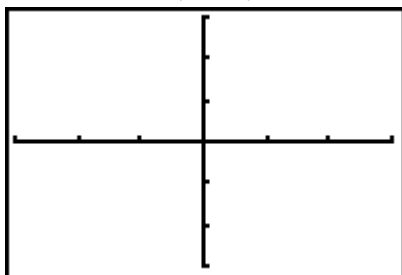
f.)  $\frac{(\sqrt{2x})^5}{(\sqrt{2x})^9}$

18.) This is the graph of a function  $y = f(x)$ .

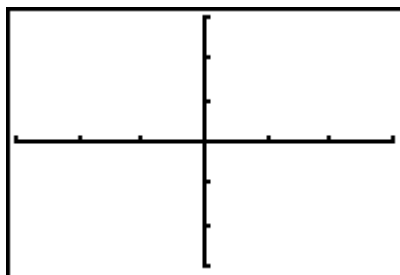


On the axes provided, please sketch the following:

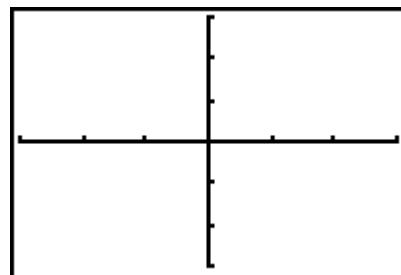
a.)  $y = |f(x)|$



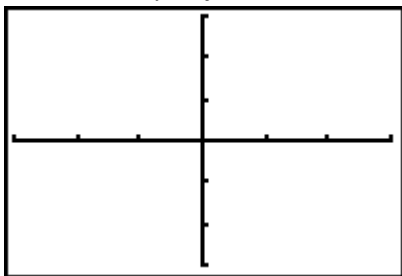
b.)  $y = f(x+1)$



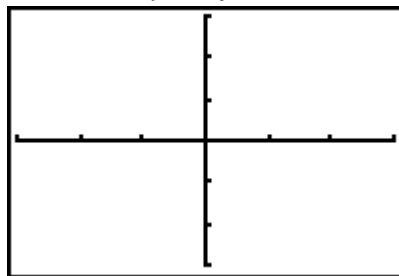
c.)  $y = -2f(x)$



d.)  $y = f(x)+1$



e.)  $y = -f(x-1)+2$

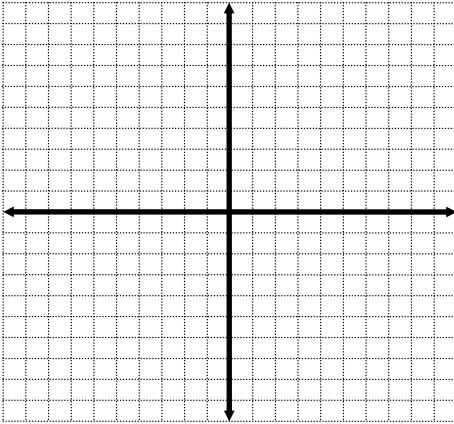


19.) a.) What are the domain and range of the function  $f(x) = 4 - x^2$  ?

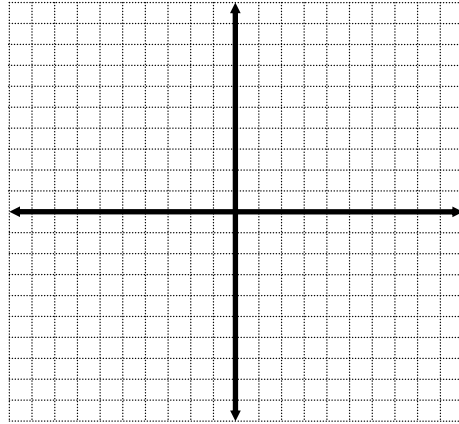
b.) What are the domain and range of the function  $f(x) = 4 - x^2$ , where  $1 \leq x < 3$  ?

20.) On the axes provided, please sketch the graph of the following:

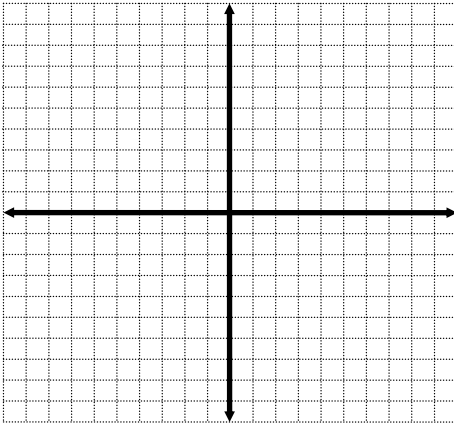
a.)  $y = -x^2 + 4x - 4$



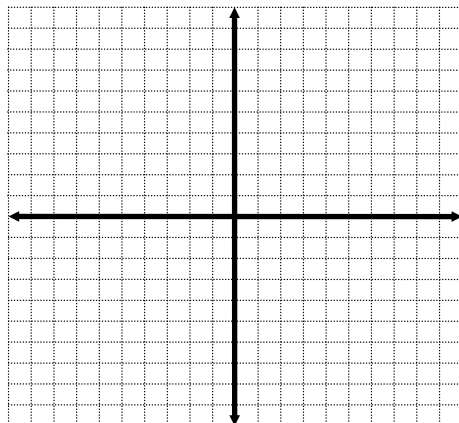
b.)  $y = \begin{cases} 2x, & \text{if } x < -1 \\ 2x^2 + x - 3, & \text{if } -1 \leq x < 2 \\ -x + 3, & \text{if } x \geq 2 \end{cases}$



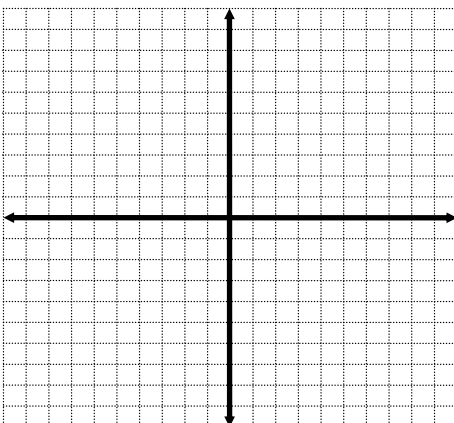
c.)  $y = \sqrt{36 - x^2}$



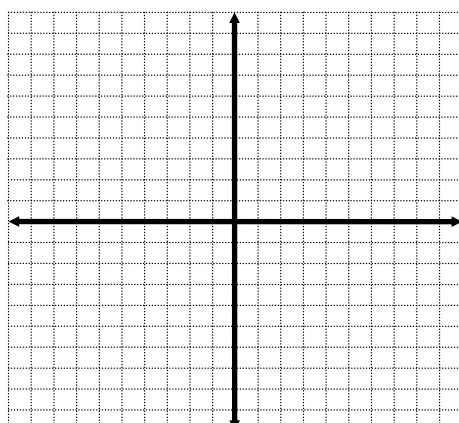
d.)  $5x^2 + 8y^2 = 80$



e.)  $y = x^{2/3}$



f.)  $y = a^x$ , where  $a > 1$



21.) Please solve each of the following for  $x$ . Identify any domain restrictions. (Note: You may need to leave your answer in logarithmic or exponential form)

a.)  $\log(x-2) = 3$

b.)  $9^x = 4$

c.)  $\ln x + \ln(2x+5) = \ln 7$

d.)  $2e^{2-x} + 5 = 12$

22.) Please simplify  $\log_2 5 - 2\log_2\left(\frac{1}{2}\right) + \log_2 1.6$  so that there are no logarithms in your answer.

23.) Please write the expression  $\log_b \left( \frac{(4x^5 - x - 1)\sqrt{x-7}}{\sqrt[3]{x^2+1}} \right)$  as a sum of logarithms.

24.) Given a box of height  $h$  and whose base dimensions are  $x$  and  $y$ , please express the *surface area AND the volume* of the box, given the following conditions:

a.) The box is closed on all sides

b.) The box has an open top

c.) The box has an open top and a square base.

25.) A 6-foot tall person is standing  $x$  feet away from a 10 foot lamppost. What is the distance  $d$  from the base of the lamppost to the end of the person's shadow, expressed as a function of  $x$ ?

26.) A piece of wire 5 meters long is to be cut into two pieces. One piece is  $x$  inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Please find an expression for the total area made up by the square and the circle as a function of  $x$ .

27.) Please determine the following (Remember . . . . No calculator!)

a.)  $\cos 0$

b.)  $\sin 0$

c.)  $\tan \frac{\pi}{2}$

d.)  $\cos \frac{5\pi}{6}$

e.)  $\sec \frac{7\pi}{4}$

f.)  $\tan 660^\circ$

g.)  $\cos^{-1} \left( \frac{\sqrt{3}}{2} \right)$

h.)  $\tan^{-1}(-1)$

28.) Given the function  $f(x) = 2 \sin \left( \frac{2}{3} x \right) + 1$ , please determine the

a.) amplitude

b.) range

c.) period

29.) Please circle the expressions that are identical:

$$\cos^2 x$$

$$(\cos x)^2$$

$$\cos(x^2)$$

$$1 + \sin^2 x$$

30.) Please circle the expressions that are identical:

$$(\sin x)^{-1}$$

$$\sin^{-1} x$$

$$\sin(x^{-1})$$

$$\frac{1}{\sin x}$$

31.) Please solve each of the following for  $\theta \in [0, 360^\circ)$

a.)  $\tan^2 \theta - 3 = 0$

b.)  $\tan \theta - \cot \theta = 0$

c.)  $\cos 2\theta = -\frac{1}{2}$

d.)  $4(1 - \sin \theta)(1 + \sin \theta) = 3$